Trajectory of primary education in Mato Grosso do Sul, Brazil (2013-2023).

Lucas Piati [a]

[a] School of Administration and Business, University of Mato Grosso do Sul, Campo Grande, Brazil, lucas.g.piati@ufms.br

Abstract: This study analyzes the structural and qualitative trajectory of public elementary education (grades 1–5) across the municipalities of Mato Grosso do Sul (MS), Brazil, between 2013 and 2023. Using data from the Basic Education Development Index (IDEB) and the IBGE School Census, twelve indicators were structured into four theoretical groups: Infrastructure, Teachers, Scale, and Performance. A Multiple Factor Analysis (MFA) was performed to examine multidimensional changes over the ten-year period. Following the Kaiser criterion, the first two dimensions were retained, accounting for 45.91% of the cumulative variance. Dimension 1 (28.08% of variance) was primarily driven by teacher effort, training adequacy, and class sizing. Dimension 2 (17.83% of variance) was strongly correlated with school size and academic performance metrics (SAEB math and IDEB scores). The spatial mapping shows a distinct shift of municipalities from the positive quadrant of Dimension 1 in 2013 to the negative quadrant in 2023, indicating a structural transformation in staffing arrangements and operational scale. The findings emphasize that long-term educational progress requires combining administrative management reforms with sustained investments in physical school infrastructure. Furthermore, this study provides a methodological framework that can be replicated in other Brazilian states to monitor educational policies. The insights aim to guide policymakers in prioritizing resource allocation to foster equitable academic environments.

Keywords: Basic Education, Educational Trajectory, Multiple Factor Analysis.

1. Introduction

Basic public education is a central component for development, and understanding its structural and qualitative progress is essential for formulating appropriate public policies [1]. This study aims to analyze the trajectory of progress in basic public education across the municipalities of the state of Mato Grosso do Sul (MS). The scope of the analysis is restricted to the early years (1st to 5th grade) and focuses exclusively on data from the municipal and state public school systems. Through the structuring of multidimensional indicators, the study investigates how human capital, infrastructure, supply scale, and productive efficiency have evolved in the region.

2. Methodology

2.1 Defining the variables

The data underlying this research were extracted from the Basic Education Development Index (IDEB) and the School Census, conducted by the Brazilian Institute of Geography and Statistics (IBGE)[2, 3]. The analyzed variables were structured into four main theoretical groups:

Group 1 - School and Technological Infrastructure (Proxy for Technological Level / Modernization): Measures the physical capital of schools through the percentage of institutions with broadband internet access (IE1), libraries or reading rooms (IE2), science and/or computer labs (IE3), and accessible facilities and pathways for students with disabilities (IE4).

Group 2 - Teacher Qualification and Effort (Proxy for Capitalization / Inputs): Evaluates the quality of human capital through the indicator of adequate teacher training (QD1) [4], the average number of students per teacher (QD2), and the percentage of teachers with temporary contracts,

acting as a proxy for precariousness or high turnover (QD3).

Group 3 - Supply Conditions and Scale (Proxy for Size): Captures the dimension of the provided service through the average number of students per class, signaling overcrowding or idleness (CO1), and the average school size based on total enrollments per establishment (CO2).

Group 4 - Performance and Efficiency (Productivity): Quantifies final learning outcomes using the IDEB synthetic score (RE1), the standardized SAEB math score (RE2), and the age-grade distortion rate, which represents the accumulated inefficiency of delayed students (RE3).

The analysis applied factorial inertia decomposition techniques to extract the principal dimensions, identifying the variance explained by each dataset and projecting the correlation of variables in a correlation circle.

2.2 Multiple Factor Analysis (MFA)

Multiple Factor Analysis (MFA) is a multivariate statistical method used to analyze datasets where a set of observations is described by multiple groups of variables[5, 6], which may be quantitative, categorical, or mixed. By applying a specific weighting scheme (dividing all variables in a group by the first singular value (the highest axial inertia) obtained from a separate Principal Component Analysis (PCA) of that specific group) MFA prevents data blocks with more variables or higher variance from dominating the global analysis. This standardization allows to simultaneously explore the underlying structure of the data, the relationships between the different groups of variables, and the spatial representation of individuals within a unified, balanced framework (Escofier & Pagès, 1994; Abdi et al., 2013).

To determine the optimal number of principal components to retain in such analyses, the Kaiser criterion is applied [7]. This rule dictates that only components with an eigenvalue greater than or equal to 1 should be retained, as they account for at least as much variance as a single original standardized variable (Kaiser, 1960).

3. Results

In the first software output, if the Kaiser criterion is applied, 3 components are chosen, which have values above 1 or very close to it (as is the case for Dim. 3), as shown below:

Tab. 1 - Eigenvalues and Variance Explained by Factorial Dimensions.

Dimension	Eigenvalue	Variance percent	Cumulative percent
Dim.1	2.25	28.08	28.08
Dim.2	1.43	17.83	45.91
Dim.3	0.94	11.71	57.62
Dim.4	0.79	9.86	67.48
Dim.5	0.64	8.00	75.48
Dim.6	0.55	6.85	82.33
Dim.7	0.40	5.05	87.38
Dim.8	0.29	3.62	91.00
Dim.9	0.26	3.26	94.26
Dim.10	0.25	3.12	97.38
Dim.11	0.17	2.17	99.55
Dim.12	0.04	0.45	100.00

Source: Authors' results using R Software

The factorial inertia decomposition generated twelve dimensions, revealing that the first two (Dim. 1 and Dim. 2) jointly explain 45.91% of the total variance. Dimension 1 has the highest eigenvalue (2.25), explaining 28.08% of the variance, while Dimension 2 (eigenvalue 1.43) accounts for 17.83% of the data explanation.

Tab. 2 - Contribution of Variable Groups to the Principal Dimensions.

Group	Dim. 1	Dim. 2	Dim. 3
Infrastructure	29.27	30.48	35.55

Teachers	35.63	7.21	8.74
Scale	30.51	24.75	28.08
Performance	4.59	37.57	27.63

Source: Prepared by the author based on IDEB and IBGE School Census data.

As seen in Table 2, the "Teachers" group demonstrated the highest percentage contribution to the formation of Dimension 1 (35.63%), followed closely by the "Scale" (30.51%) and "Infrastructure" (29.27%) groups. In Dimension 2, the predominant weight occurred in the "Performance" (37.57%) and "Infrastructure" (30.48%) groups.

Tab. 3 - Individual Variable Contributions to the Principal Dimensions (%)

Variable	Dim. 1	Dim. 2	Dim. 3
% Schools with Internet	6,27	8,03	2,99
% Schools with Library	0,03	13,60	11,20
% Schools with Science/Computer Lab	9,10	6,24	19,87
% Schools with Accessibility	13,87	2,61	1,50
Teacher Training Adequacy	12,64	0,19	2,91
% Teachers with Low Effort / Temporary Contract	14,18	0,31	3,24
Average Students per Teacher	8,81	6,70	2,59
Average Students per Class	27,53	0,00	0,03
Average School Size	2,98	24,75	28,05
IDEB Score	4,56	15,60	1,50
SAEB Math Score	0,01	21,81	0,11
Age-Grade Distortion Rate	0,02	0,15	26,03

Source: Prepared by the author based on IDEB and IBGE School Census data.

When analyzing the individual weight of the variables (Table 3), the average number of students per class showed the strongest contribution to Dimension 1 (27.53%), along with the percentage of low-effort/temporary contract teachers (14.18%) and adequate training (12.64%). Concurrently, the variables that most drove Dimension 2 were the average school size (24.75%) and the standardized SAEB math score (21.81%).

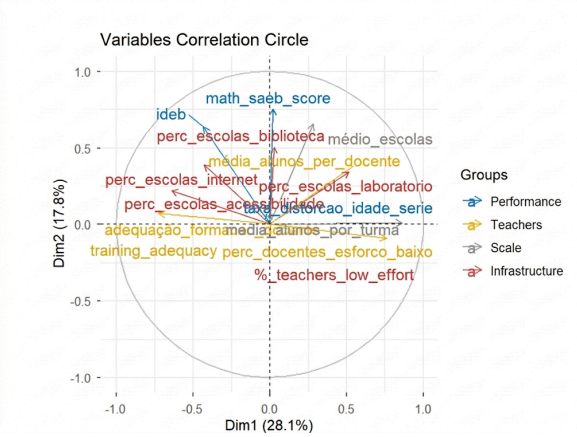


Fig. 1 - Graph of Variables Correlation Circle.

The correlation circle graph (Fig. 1) confirms that variables such as the IDEB score, the SAEB math score, and the presence of libraries share a strong positive correlation on the Dimension 2 axis. The "Educational Trajectory of MS (2013 - 2023)" graph (Fig. 2) illustrates a clear migration of the municipalities' coordinates over time. The observations for the year 2013 are heavily concentrated in the positive pole of Dimension 1 (to the right of the axis), while the 2023 observations dominantly cluster in the negative pole of Dimension 1 (to the left of the axis).

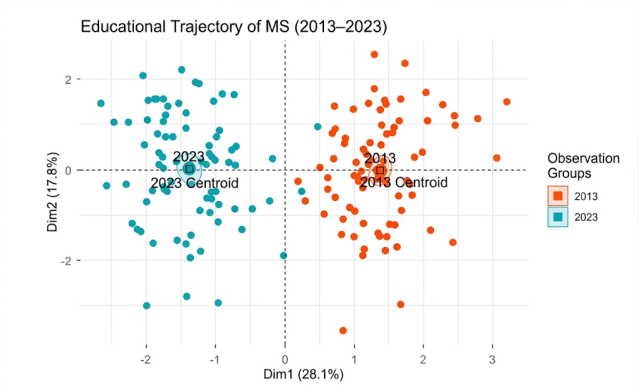


Fig. 2 - Trajectory Graph.

4. Conclusions

The significant temporal separation observed in the trajectory graph (Fig. 2) shows that the public basic education system in Mato Grosso do Sul underwent substantial and systematic changes over the ten-year interval (2013-2023). The fact that the shift occurs primarily along Dimension 1 suggests that the main transformation in early-years schools is

linked to scale organization and the profile of the teaching staff. Since Dimension 1 is strongly defined by variables related to teacher contracts, training adequacy, and class size, the migration of municipalities along this axis points to fundamental restructurings in hiring policies and student allocation.

Additionally, the clustering of variables in Dimension 2 reinforces the close interdependence between infrastructure modernization—especially regarding school size and physical resources like libraries—and the quantitative metrics of final student productivity (SAEB and IDEB). The data demonstrate that tangible investments in the physical capital of the school environment structurally align with learning proficiency. Overall, the measured educational progress in Mato Grosso do Sul illustrates the need to concomitantly align reforms in operational conditions (school size and teacher stability) with continuous improvements in technological and physical capital [2, 3].

5. References

[1] UNESCO. *Global Education Monitoring Report: Inclusion and education for all*. UNESCO Publishing, Paris; 2020; 500 p.

[2] IBGE. *Censo Escolar e Indicadores do IDEB: Dados Educacionais Municipais de Mato Grosso do Sul*. Instituto Brasileiro de Geografia e Estatística, Rio de Janeiro; 2023; 120 p.

[3] INEP. *Notas Estatísticas do Censo da Educação Básica 2023*. Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira, Brasília; 2024; 68 p.

[4] INEP. *Nota Técnica nº 020/2014: Indicador de Adequação da Formação Docente*. Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira, Brasília; 2014; 15 p.

[5] Escofier B., Pagès J. Multiple Factor Analysis (AFMULT Package). *Computational Statistics & Data Analysis*. 1994; 18(1):121-140.

[6] Abdi H., Valentin D. Multiple Factor Analysis. *Encyclopedia of Measurement and Statistics*. 2007; 1(1):657-663.

[7] Kaiser H. F. The application of electronic computers to factor analysis. *Educational and Psychological Measurement*. 1960; 20(1):141-151.